An optical implementation of a space-time-code for enhancing the tolerance of systems to polarization dependent loss

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We propose a new space-time coding scheme designed to increase the tolerance of fiber-optic communications systems to polarization dependent loss (PDL). A notable increase in the tolerable amount of average link PDL is achieved without affecting the complexity of the overall optical communications link. Other advantages include seamless integration with the broadly deployed blind equalization modules relying on the constant modulus algorithm.

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Increasing the tolerance of coherent polarization multiplexed networks to polarization dependent loss (PDL) has been recognized as an important problem in optical communications. Space-time coding [1] is a natural approach, and the Golden and Silver codes that were studied in this context so far [2, 3] were predicted to increase the tolerable amount of PDL by approximately 2 dB (assuming standard PDL margin levels of 1 to 2 decibels). The main drawback of these codes, which were designed for the wireless communications environment, is the decoder's complexity which notably increases the overall complexity of the receiver. Another disadvantage is incompatibility with blind equalizers based on the constant modulus approach [4], which are now ubiquitously deployed in fiber-communications.

In this Letter we consider a new space-time coding scheme that increases the tolerable PDL of a fiber optic system by approximately 1dB and with practically no effect on the overall complexity. A convenient feature of the proposed scheme is that the encoder can be implemented optically by the use of a single birefringent delay element. We do not argue that the optical implementation is preferable from a practical standpoint, but it is useful for illustrating the inspiration for this code and for understanding its principle of operation. Although the performance of the proposed space-time-code is inferior to that of the Silver or Golden codes, its advantage is in the decoding simplicity and in the fact that it can be readily combined with standard equalization techniques with no effect on their complexity.

In order to explain the principle of operation we assume the optical implementation of Fig. 1a, where a birefringent element of differential group delay τ is aligned such that its principle axes are at a 45 degrees angle between the horizontal (H) and vertical (V) axes along which the two data streams are modulated. The effect of the birefringent delay element is conveniently represented in Stokes space, where the H and V polarizations are mapped to the points $\pm\hat{S}_1$. Transmission through

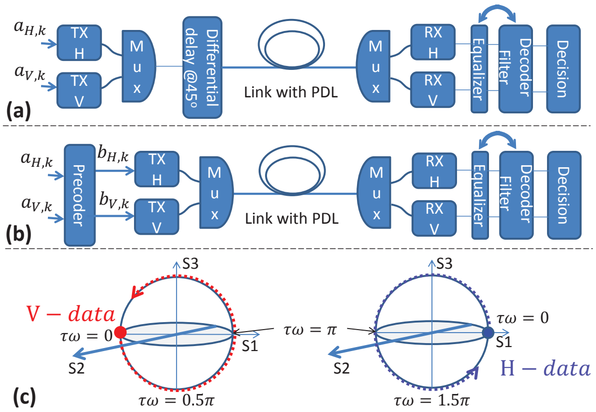

Fig. 1. (a) The optical implementation contains a birefringent element introducing a differential delay τ between the polarization axes at 45 and -45 degrees with respect to H and V. (b) The digital implementation is a space-time code. (c) The data transmitted over the H and V polarizations is smeared over a large circle on the Poincaré sphere, resulting in averaging out the effect of PDL.

the birefringent delay element corresponds to frequency dependent rotation about the axis $\hat{S}_2$ (see Fig. 1 c). Thus the spectral content of the the H and V polarized channels is smeared on a great circle of the Poincaré sphere. Assuming that τ is large enough to ensure uniform smearing, the signal will always be equally affected by the maximum and minimum attenuation axes of PDL in the subsequent optical link.

The equivalence to digitally implemented space-time coding is straightforward when noting that the signals in the H and V polarizations immediately following the birefringent delay element, are given by

$$E_H(t) = \frac{s_H(t) + s_V(t)}{2} + \frac{s_H(t-\tau) - s_V(t-\tau)}{2} \quad (1)$$

$$E_V(t) = \frac{s_H(t) + s_V(t)}{2} - \frac{s_H(t-\tau) - s_V(t-\tau)}{2}, \quad (2)$$

where $s_H(t)$ and $s_V(t)$ represent the signals produced by the two modulators. Naturally, if τ is equal to an integer multiple of the symbol duration T_s, then the same fields can be generated by precoding the digital data symbols (see Fig. 1b). Thus, assuming $\tau = mT_s$, the data symbols $b_{H,k}$ and $b_{V,k}$ launched in the k-th symbol duration into the H and V polarized channels, respectively, are

$$b_{H,k} = \frac{a_{H,k} + a_{V,k}}{2} + \frac{a_{H,k-m} - a_{V,k-m}}{2} \tag{3}$$

$$b_{V,k} = \frac{a_{H,k} + a_{V,k}}{2} - \frac{a_{H,k-m} - a_{V,k-m}}{2}, \tag{4}$$

where $a_{H,k}$ and $a_{V,k}$ are the data symbols prior to precoding. Equations (3-4) constitute a space-time trellis code (STTC) [5] requiring appropriate trellis decoding for maximum likelihood operation [6]. Such decoding is fairly complex in optical communications rates and as the present work is motivated by alleviating complexity, we do not consider it here. Instead, taking advantage of the comparatively moderate fading caused by PDL in optical systems (the average PDL of a link rarely exceeds few decibels), our proposed scheme relies on a very simple, albeit suboptimal decoding scheme. We assume that the the receiver consists of a digital equalizer followed by a fixed decoder implementing the relations

$$a_{H,k} = \frac{b_{H,k} + b_{V,k}}{2} + \frac{b_{H,k+m} - b_{V,k+m}}{2} \tag{5}$$

$$a_{V,k} = \frac{b_{H,k} + b_{V,k}}{2} - \frac{b_{H,k+m} - b_{V,k+m}}{2}. \tag{6}$$

which invert Eqs. (3-4). In the case of adaptive equalization, the feedback to the equalizer comes from the output of the fixed decoder, as indicated by the arrows in Figs. 1a and 1b. In this configuration, the complexity (i.e. number of taps) in the adaptive equalization block is not affected by the delay introduced by the space-time code.

In what follows we evaluate the performance of the proposed scheme in the case of single-carrier quadrature phase-shift-keying (QPSK) transmission using Nyquist shaped pulses of rectangular bandwidth $1/T_s$, with T_s denoting the symbol duration. The metric that we use for quantifying the PDL tolerance is the system SNR margin that is required to ensure that the probability of a system outage due to PDL remains lower than an outage probability of 4×10^{-5} (or is 20 minutes per year). We evaluate the performance of the proposed scheme under the assumption of a zero-forcing (ZF) receiver [7] and validate the results in simulations of the constant modulus algorithm (CMA), followed by a decision driven least mean squares (DD-LMS) filter as proposed in [4]. The ZF receiver is not optimal, as it simply inverts the transfer matrix of the channel while ignoring the partly polarized nature of the noise. Nonetheless, as we showed in [7], its performance is very reasonable when practically relevant values of PDL are assumed. Our choice of the ZF receiver is driven mainly by its computational efficiency, which allows us to simulate a very large number of random system realizations.

In all of our simulations, the transmission link is assumed to be linear and chromatic dispersion is assumed to be perfectly compensated. In addition, we assume that the PMD of the link is negligible, so that the received signal is impaired only by PDL and by amplifier noise (which is partly polarized due to the presence of PDL [9]). The neglect of PMD is immaterial to our study since with typically encountered PMD levels, its effect on PDL tolerance is negligible (unless PMD is introduced intentionally as in [10]). The link is simulated as in [9] with 10 statistically independent PDL sections, ensuring that the overall PDL statistics is consistent with [11].

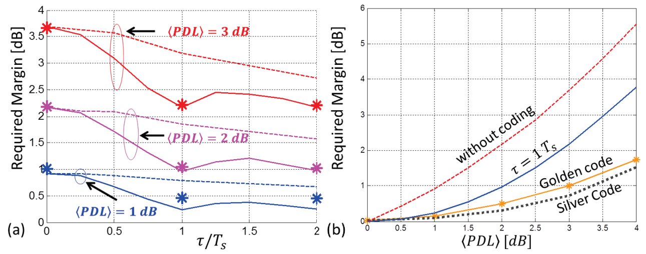

Fig. 2. (a) Required link margin as a function of the coding delay. The solid curves correspond to the proposed scheme with a zero-forcing receiver and the stars indicate the CMA receiver results. The CMA receiver was only simulated in the case of integer values of τ/T_s due to the long computation times associated with the algorithm convergence. The dashed curves show the results that would be obtained if instead of the space-time code proposed here, PMD with average DGD equal to τ were distributed along the link as in [10]. (b) The required margin as a function of the mean PDL in the uncoded case and with the proposed code with $\tau = T_s$. The results of the Golden and Silver codes were copied from [3] for reference.

The solid curves in Fig. 2a show the required PDL margin as a function of the coding delay introduced by the differential delay element shown in Fig. 1a. These curves were obtained based on 10^6 random link realizations and under the assumption of a ZF receiver. The dashed curves are shown for reference and they represent the PDL margin that would be required if instead of the proposed scheme, random PMD with average differential delay τ, were distributed randomly along the link as considered in [10]. This result too was obtained under the assumption of a ZF receiver. We will return to the comparison between the dashed and the solid curves in what follows. Due to the very long computation times associated with the simulation of the CMA receiver, the results corresponding to CMA were obtained only in the case of integer τ/T_s values. These are shown by the stars in Fig. 2b, which are very close to the solid curves in all cases, thereby validating the ZF results. Figure 2a shows the required margin as a function of the average PDL for the uncoded system and with the proposed code implemented with $\tau = T_s$ and ZF equalization. The results of the Golden and Silver codes (with optimal decoding) were reproduced from [3] for comparison. The inferiority of the proposed scheme is negligible when the mean PDL is 1dB or lower and grows to approximately 1.5dB for 3dB of average PDL. Although this is a notable difference, there is still approximately 1.5dB improvement relative to the uncoded scheme in this case.

Focusing on the solid curves, the required PDL margin is seen to reduce with the coding delay until it reaches its minimum value when $\tau = T_s$. Then it returns to the same minimum value repeatedly when τ/T_s is an integer. The reason for this behavior is that the assumption of Nyquist-shaped pulses implies a rectangular spectrum, which for integer τ/T_s values is distributed evenly on a great circle of the Poincaré sphere. In this situation the frequency averaged effect of PDL on the signal is independent of the PDL vector's orientation and optimal averaging occurs. The averaging principle is illustrated in Fig. 3, where the cumulated probability of the SNR reduction caused by the presence of PDL is plotted for mean PDL values of 1 and 2 dB. The leftmost solid curve represents the SNR distribution corresponding to the polarization channel (H or V) that experiences the largest PDL penalty. Similarly the rightmost solid curve represents the SNR distribution seen by the best of the two channels. The dashed curve between them shows the distribution of the average SNR of the two polarization channels. The diamonds and the circles show the SNR distributions of the two polarization channels when the proposed coding scheme is applied with $\tau = T_s$, and the overlap with the dashed curve demonstrates the excellent averaging that takes place.

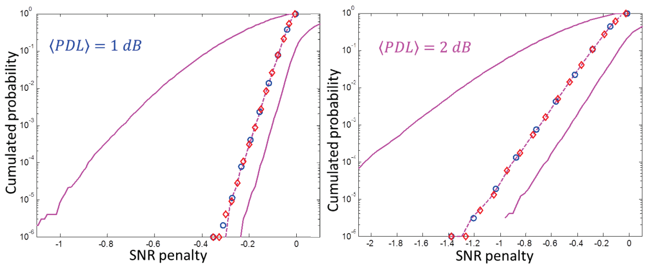

Fig. 3. The cumulated distribution of the SNR when only the worst of the two polarization channels is transmitted (leftmost curve), the best of the two channels is transmitted (rightmost curve) and the cumulated distribution of the average SNR (middle dashed curve). The dots on the middle curve represent the distribution of the overall SNR when both channels are transmitted and the proposed coding scheme is implemented with a coding delay of one symbol. The fact that the circles overlap with the middle curve indicate the perfect averaging that takes place. This curve was produced under the assumption of a ZF receiver.

We now return to the comparison between the coding scheme proposed here and the case represented by the dashed curves in Fig. 2 where large values of PMD are distributed along the link [10]. Although in both cases the signal is manipulated by a unitary transformation that involves the introduction of a differential group delay (DGD), the principles of operation are distinctly different. In the presence of PMD, the PDL vector itself (both direction and modulus) becomes frequency dependent and its correlation bandwidth is known to be given by π/τ [12]. Therefore, achievement of effective frequency averaging within the bandwidth of the information carrying signal requires that τ is well in excess of the symbol duration T_s. In the scheme that we propose here, the differential delay element (in the equivalent optical implementation of Fig. 1a) is placed before the link and introduces no frequency dependence to the magnitude of the PDL. Instead, its only effect is to distribute the data over a continuum of polarization states, thereby introducing diversity. The thin dashed curves in Fig. 2a show the required PDL margin in the case of the PMD-based scheme of [10]. These curves reduce monotonically and can be shown to intersect with the solid curves representing the present scheme only when $\tau/T_s \gg 1$. Thus, although the presence of PMD will eventually produce better PDL tolerance than the proposed coding scheme, it implies very large delays and hence the need for fairly complex equalization.

To conclude we have introduced a space-time coding scheme that increases the PDL tolerance of systems with practically no complexity overhead. The benefit in terms of the tolerable PDL is estimated to be in the vicinity of 1dB. This can be extracted from Fig. 2 by noting that with $\tau = T_s$ the required PDL margin is similar to the margin required with 1dB less of PDL without coding (i.e. with $\tau = 0$). An important advantage of the proposed scheme is that it can be seamlessly integrated with available equalization technology, including the ubiquitous CMA-based blind equalized.

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